STUDIES ON THE VORTICES OF THE ATMOSPHERE OF THE EARTH.

By Prof. FRANK H. BIGELOW. Dated May 6, 1908.

III.—THE TRUNCATED DUMB-BELL VORTEX ILLUSTRATED BY THE ST. LOUIS, MO., TORNADO OF MAY 27, 1896.

On the 27th of May, 1896, a large, violent tornado past thru the midst of the city of St. Louis, Mo., doing much damage, and displaying all the features which characterize this class of storms. The meteorological conditions were reported by Dr. H. C. Frankenfield, Local Forecast Official, and an account was published in the MONTHLY WEATHER REVIEW for March, 1896, from which the data used in this paper have been extracted, the same being confirmed by comparison with original records. A study of the mechanical forces exerted upon the buildings which were wrecked, and the St. Louis Bridge which was injured, was made by Mr. Julius Baier, and reported in the Journal of the American Society of Civil Engineers, January and June, 1897. An illustrated phamphlet, by Martin Green, St. Louis, Mo., 1897, gives a graphic description of the numerous destructive effects of the storm. In Table 30 will be found a summary of the meteorological conditions for reference. The vortex proper, apparently lasted from 6 to 6:30 p. m., at the Weather Bureau station, and was central about 6:15 p. m. The barometer oscillated considerably, but the pressure at the station, 29.00 inches, or 737 millimeters (not reduced to sea level), was assumed for the outer radius, ϖ_1. The pressures, temperatures, wind directions, and wind velocities are taken from the automatic records of the Weather Bureau, of which transcripts will be found in Doctor Frankenfield's article.

TABLE 30.—*The meteorological data at St. Louis, Mo., May 27, 1896.*

Time—90th meridian.	Station pressure.	Temperature.	Relative humidity.	Wind.		State of sky.
				Velocity.	Direction.	
	Inches.	°	%	*mi.p.h.*		
8:00 a. m	29.32	70	94	8	s.	3 S.-Cu.
Noon	29.25	78	....	13	sw.	
2:00 p. m	29.22	84	....	7	se.	8 A.-S.
3:00 p. m	29.17	84	....	10	se.	
4:00 p. m	29.12	84	....	19	se.	10 A.-S.
4:30 p. m	29.10	84	....	18	se.	
5:00 p. m	29.05	81	....	25	se.	10 Cu.
5:10 p. m	29.07	82	....	24	se.	10 Cu.-N.
5:20 p. m	29.07	81	....	22	se.	10 N.
5:30 p. m	29.05	80	....	23	se.	10 N.
5:40 p. m	29.05	79	....	30	se.	10 N.
5:50 p. m	29.04	77	100	19	e.	10 N.
6:00 p. m	28.97	72	....	44	ne.	10 N.
6:10 p. m	28.97	....	....	38	se.	10 N.
6:20 p. m	28.74	....	....	80	nw.	10 N.
6:30 p. m	29.14	....	....	34	n.	10 N.
6:40 p. m	29.14	....	....	16	ne.	10 N.
6:50 p. m	29.10	....	....	7	ne.	10 N.
7:00 p. m	29.05	67	....	17	e.	10 N.
8:00 p. m	29.16	65	100	10	n.	
9:00 p. m	29.18	....	....	....	....	
10:00 p. m	29.14	66	....	....	....	

The vortex was central about 6:15 p. m.

An aneroid barometer, read by the son of Mr. Klemm, on the south side of Lafayette Park near the center of the storm, indicated 680 millimeters = 26.78 inches. This checks in reading 677 millimeters obtained by the vortex rings.

TABLE 31.—*Adopted pressure on the center of the path of the tornado.*

Time.	Pressure.		Position.
	Inches.	*mm.*	
6:00 p. m....	29.00	737	Edge of vortex, ϖ_1
	28.62	727	ϖ_2
	28.23	717	ϖ_3
	27.84	707	ϖ_4
	27.44	697	ϖ_5
	27.05	687	ϖ_6
6:15 p. m....	26.65	677	Center of vortex, ϖ_7

THE DATA FOR COMPUTING THE VORTEX.

The tornado past over the city from west to east, and apparently the center of the vortex crost Lafayette Park, whence it proceeded to the great bridge spanning the Mississippi River. The Weather Bureau office is about seven blocks north of the park, and it was estimated that the vortex was about one and one-fourth miles wide. It is necessary to determine at what chord the instruments of the Weather Bureau crost the vortex tubes, so that the records may be suitably interpreted. By a careful study of the De Witte typhoon, in which case the meteorological data sufficed to determine several of the individual isobars, from which the ratio of the successive radii could be found, it was possible to construct a vortex diagram suitable to the atmosphere. This same scale of vortex was adopted for the St. Louis tornado, as the data to construct a complete vortex independently were lacking, and it was only necessary to find the pressure and the wind direction and velocity at a few points. The pressure, 29.00 inches (737 millimeters) was taken for ring ϖ_1, and the successive rings were given a pressure diminishing by 10 millimeters, until the seventh ring ϖ_7, was found with a pressure of 26.65 inches (677 millimeters) near the center of the vortex.

The following note appears in Doctor Frankenfield's paper, added June 23, 1896:

I have just learned of the height of the barometer, within a reasonable degree of accuracy, in or very near the center of the track of the tornado at the time it moved thru Lafayette Park. It was in this park that the storm was at its height. An aneroid barometer, with a metrical scale, was brought to me to be reset, and I was informed that it was the property of the widow of the late Richard Klemm, ex-Park Commissioner of the city. The family live on Missouri avenue, immediately fronting the park, and a son of Mr. Klemm read the barometer as the storm struck their place. He called the attention of his mother to the remarkably low reading, 680 millimeters, or 26.78 inches. Allowing for difference in elevation and reduction to sea level, this would indicate a reduced reading of 27.30 inches, or 2.05 inches lower than observed at this office.

If the barometric pressure was 26.65 inches near the center, the pressure at the Weather Bureau office would be 26.65+2.05=28.70 inches. As the observation gave 28.74 inches, we may suppose that the office passed near or within the ϖ_2 circle. I have taken it somewhat within this circle, because the Klemm house was a little south of the central line of the vortex as marked by the débris, and it is probable that its position is between circles ϖ_6 and ϖ_7.

TABLE 32.—*Table of observed wind velocities near the vortex center.*

Time, p. m.	Wind.	
	Velocity.	Direction.
	mi.p.h.	
5:50–5:55	37	e.
5:55–6:00	44	ne.
6:00–6:05	28	se.
6:05–6:10	38	se.
6:10–6:15	60	nw.
6:15–6:20	80	nw.
6:20–6:25	44	ne.

It appears from Table 32 that the wind velocity between 5:50 and 6:05 p. m. averaged about 33 miles per hour, and that it averaged 56 miles per hour from 6:05 to 6:25 p. m. There were great oscillations in the wind velocity, the maximum for less than one minute being 120 miles per hour at 6:18 p. m. A study of the wind directions in all possible detail shows that the wind cut the isobars at about 30°, so that the angle $i=-30°$, whence, by the formula $az=90°+i$, the angular altitude $az=60°$. These values are adopted for the computations on the vortex.

On the isobar whose radius$=\varpi_1$, $q_1=33$ miles per hour.
$i=-30°$.

On the isobar whose radius $= \varpi_2$, $q_2 = 56$ miles per hour.
$i = -30°$.

Adopting the values $q_1 = 33$ miles per hour $= 15$ meters per second on ϖ_1, and $q_2 = 56$ miles per hour $= 25$ meters per second on ϖ_2, we obtain the following:

Tube.	(1)	(2)	
$\varpi =$	960 m.	600 m.	Adopted radii.
$u = q \sin i$.........	-7.5	-13.0	Adopted radial velocities.
$v = q \cos i$................ ..	12.5	21.6	Adopted tangential velocities.

It is intended to compute the average vortex at the outset, and then to discuss it by applying the proper formulas and the divergences between the angles and velocities in the mean vortex and that occurring in nature. The mode of constructing this mean vortex from a few available observations will be given in detail. The formulas for this type of vortex are repeated here for convenience.

GENERAL FORMULAS.

$$v\varpi = \quad a\psi \quad = \quad A a \varpi^2 \sin az.$$

$$u = -\frac{1}{\varpi}\frac{\partial \psi}{\partial z} = -A a \varpi \cos az.$$

$$v = \quad \frac{a\psi}{\varpi} \quad = \quad A a \varpi \sin az.$$

$$w = +\frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} = +2A \sin az.$$

The first step is to scale from the diagram on the adopted radius, $\varpi_1 = 960$ meters, the length of the other radii.

TABLE 33.—*Computation of the mean* $(v\varpi) = a\psi$.

Term.	Number.	Logarithm.	Term.	Number.	Logarithm.
ϖ_1	960	2.98227	ϖ_2	600	2.77815
v_1	13	1.11394	v_2	21	1.32222
u_1	8	....	u_2	13	....
$(v\varpi)_1$	.....	4.09621	$(v\varpi)_2$	.....	4.10037
Mean $(v\varpi)$	12 554	4.09879			

Then take log ϖ_n and the successive differences, log ρ = log ϖ_n − log ϖ_{n+1}; compute the mean log $\rho = 0.20546$; construct log $(v\varpi)_1 = 4.09621$ and log $(v\varpi)_2 = 4.10037$; the mean log $(v\varpi)$ = log $a\psi = 4.09879$, and this is the constant to be laid at the basis of the vortex. In this case we assumed $(v\varpi)_1 = 13 \times 960$ and $(v\varpi)_2 = 21 \times 600$, taking approximate values. If more observed velocities are at hand, the value of log $a\psi$ = constant can be made more accurate. Finally, from log ϖ_1 we compute log ϖ_2, log ϖ_3, etc., by subtracting 0.20546 in succession, whence the values in Table 34.

TABLE 34.—*Computation of the mean* log ρ *and the radii* ϖ_n.

B.	ϖ	log ϖ	log ρ	log ϖ_n	ϖ_n	Tubes.
Mm.	*Meters.*				*Meters.*	
737	960	2.98227		2.98227	960.0	ϖ_1
			0.20412			
727	600	2.77815		2.77681	598.2	ϖ_2
			0.20412			
717	375	2.57403		2.57135	372.7	ϖ_3
			0.19382			
707	240	2.38021		2.36589	232.2	ϖ_4
			0.23408			
697	140	2.14613		2.16043	144.7	ϖ_5
			0.19189			
687	90	1.95424		1.95497	90.2	ϖ_6
			0.21388			
677	55	1.74036		1.74951	56.2	ϖ_7
			0.19629			
667	35	1.54407		1.54405	35.0	ϖ_8
		Mean log ρ =	0.20546			

In order to determine the angular constant a, it was assumed that the effective cloud forming the tornado was 1,200 meters above the surface, and that consequently 600 meters had been cut off from the vortex tube, because $az = 60°$ was below the surface, since $i = -30°$ at the surface on the horizontal plane as shown in Chart IX, fig. 6.

$$\text{Hence, } a = \frac{180°}{1200+600} = \frac{180°}{1800} = 0.10°.$$

We compute the velocity components on the plane $az = 60°$ as follows:

For $az = 60°$,

log sin az = log sin 60° = 9.93753
log cos az = log cos 60° = 9.69897

For $a = 0.10°$,

log a = log 0.10° = 9.00000

hence,

log a sin az = 8.93753
log a cos az = 8.69897

which are constants for $az = 60°$, or $i = -30°$.

TABLE 35.—*Computation of A, u, v, w for each radius* ϖ_n, *az* = 60°.

Term.	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7
log ϖ	2.98227	2.77681	2.57135	2.36589	2.16043	1.95497	1.74951
log v	1.11652	1.32198	1.52744	1.73290	1.93836	2.14382	2.34928
v	13.1	21.0	33.7	54.1	86.8	139.3	223.5
log $a\varpi$ sin az	1.91980	1.71434	1.50888	1.30342	1.09796	0.89250	0.68704
log A	9.19672	9.60764	0.01856	0.42948	0.84040	1.25132	1.66224
A	0.1573	0.4052	1.0437	2.6883	6.9247	17.8371	45.9450
log u	−0.87796	−1.08342	−1.28888	−1.49434	−1.69980	−1.90526	−2.11072
u	−7.6	−12.1	−19.5	−31.2	−50.1	−80.4	−129.0
log w	9.43528	9.84620	0.25712	0.66804	1.07896	1.48988	1.90080
w	0.27	0.70	1.81	4.66	11.99	30.89	79.58
log $Aa\varpi$	1.17899	1.38445	1.58991	1.79537	2.00083	2.20629	2.41175
$Aa\varpi$	15.10	24.24	38.90	62.43	100.19	160.80	258.08

The successive values of log ϖ_n in Table 35 are formed by subtracting 0.20546; the values of log v are formed by adding 0.20546 to log v_1, or by subtracting the successive log ϖ_n from log $a\psi = 4.09879$. The values of log $a\varpi$ sin az are formed by adding 8.93753 to the successive log ϖ_n. To obtain log A_n subtract the successive values of log $a\varpi_n$ sin az from the successive log v_n. To compute the values of log u_n add log a cos az, log ϖ, and log A. The values of log w_n are found from $2A$ sin az.

TABLE 36.—*The logarithms of quantities useful in computing* ϖ_1, u_1, v_1, w_1, *on the 10-degree levels.*

Angle az.		sin az.	Diff.	½ Diff.	cos az.	$Aa\varpi$.	zA.
°	°						
az = 0	180	−∞	....	....	0.00000	∞	
10	170	9.23967			9.99335	1.52792	9.49775
			0.29438	0.14719			
20	160	9.53405			9.97299	1.38073	9.49775
			0.16492	0.08246			
30	150	9.69897			9.93753	1.29827	9.49775
			0.10910	0.05455			
40	140	9.80807			9.88425	1.24372	9.49775
			0.07618	0.03809			
50	130	9.88425			9.80807	1.20563	9.49775
			0.05328	0.02664			
60	120	9.93753			9.69897	1.17899	9.49775
			0.03546	0.01773			
70	110	9.97299			9.53405	1.16126	9.49775
			0.02036	0.01018			
80	100	9.99335			9.23967	1.15108	9.49775
			0.00665	0.00333			
90	90	0.00000			−∞	1.14775	9.49775

The difference of the successive values of log ϖ is equal to − log ρ; of log v is + log ρ; of log A is + 2 log ρ; of log u is + log ρ, and of log w is + 2 log ρ. Checks on the computation can be readily formed from these precepts.

COMPUTATION OF ϖ_n, u_n, v_n, w_n ON OTHER LEVELS.

The computation of the radii, the radial, tangential, and vertical velocities on other levels, as for $az = 60°, 70°, 80°, 90°, \ldots\ldots 180°$, is accomplished by using the proper trigonometric functions as called for by the formulas. In extending the logarithms to the 10-degree levels, Table 36 will be found useful. Since $a\psi = Aa\varpi^2 \sin az$, we have

$$(54) \qquad \varpi = \left(\frac{a\psi}{Aa \sin az}\right)^{\frac{1}{2}}$$

as the formula for computing $\log \varpi$.

If ϖ is computed for the level $az = 60°$, it can be extended to the other levels by using the column $\frac{1}{2}$ diff. $= \frac{1}{2}[\log \sin az - (\sin az \pm 10°)]$.

Table 37 contains the values of $\log \varpi$ and ϖ, the radii of the different levels of the seven vortex tubes.

TABLE 37.—*Computation of $\log \varpi$ and ϖ, for each tube at successive altitudes.*

Value of $\log \varpi$.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	∞	∞	∞	∞	∞	∞	∞
	170	3.33120	3.12574	2.92028	2.71482	2.50936	2.30390	2.09844
	160	3.18401	2.97855	2.77309	2.56763	2.36217	2.15671	1.95125
	150	3.10155	2.89609	2.69063	2.48517	2.27971	2.07425	1.86879
	140	3.04700	2.84154	2.63608	2.43062	2.22516	2.01970	1.81424
	130	3.00891	2.80345	2.59799	2.39253	2.18707	1.98161	1.77615
60°	120	2.98227	2.77681	2.57135	2.36589	2.16043	1.95497	1.74951
70	110	2.96454	2.75908	2.55362	2.34816	2.14270	1.93724	1.73178
80	100	2.95436	2.74890	2.54344	2.33798	2.13252	1.92706	1.72160
90	90	2.95103	2.74557	2.54011	2.33465	2.12919	1.92373	1.71827

The radius $\varpi = \left(\frac{a\psi}{Aa \sin az}\right)^{\frac{1}{2}}$.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	∞	∞	∞	∞	∞	∞	∞
	170	2143.9	1335.8	832.3	518.6	323.1	201.3	125.4
	160	1527.6	951.8	593.0	369.5	230.2	143.5	89.4
	150	1263.4	787.2	490.5	305.6	190.4	118.6	73.9
	140	1114.3	694.3	432.5	269.5	167.9	104.6	65.2
	130	1020.7	636.0	396.3	246.9	153.8	95.9	59.7
60°	120	960.0	598.1	372.7	232.2	144.7	90.2	56.2
70	110	921.6	574.2	357.8	222.9	138.9	86.5	53.9
80	100	900.2	560.9	349.5	217.8	135.7	84.5	52.7
90	90	893.4	556.6	346.8	216.1	134.6	83.0	52.3

THE VELOCITIES IN THE ST. LOUIS TORNADO.

TABLE 38.—*The computation of radial velocities u, for each tube and altitude.*

Values of $\log u$.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	∞	∞	∞	∞	∞	∞	∞
	170	1.52127	1.72673	1.93219	2.13765	2.34311	2.54857	2.75403
	160	1.35372	1.55918	1.76464	1.97010	1.17556	2.38102	2.58648
	150	1.23580	1.44126	1.64672	1.85218	2.05761	2.26310	2.46856
	140	1.12797	1.33343	1.53889	1.74435	1.94981	2.15527	2.36073
	130	1.01370	1.21916	1.42462	1.63008	1.83554	2.04100	2.24646
60°	120	0.87796	1.08342	1.28888	1.49434	1.69980	1.90526	2.11072
70	110	0.69531	0.90077	1.10623	1.31169	1.51715	1.72261	1.92807
80	100	0.39075	0.59621	0.80167	1.00713	1.21259	1.41805	1.62351
90	90	—∞	—∞	—∞	—∞	—∞	—∞	—∞

Values of the radial velocity, $u = -Aa\varpi \cos az$.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$az = 180°$	∞	∞	∞	∞	∞	∞	∞
170	33.2	53.3	85.5	137.3	220.3	353.6	567.6
160	22.6	36.2	58.2	93.4	149.8	240.4	385.9
150	17.2	27.6	44.3	71.2	114.2	183.3	294.1
140	13.4	21.6	34.6	55.5	89.1	143.0	229.5
130	10.3	16.6	26.6	42.7	68.5	109.9	176.4
120	7.6	12.1	19.5	31.2	50.1	80.4	129.0
110	5.0	8.0	12.8	20.5	32.9	52.8	84.7
100	2.5	4.0	6.3	10.2	16.3	26.2	42.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
80	— 2.5	— 4.0	— 6.3	—10.2	—16.3	—26.2	— 42.0
70	— 5.0	— 8.0	—12.8	—20.5	—32.9	—52.8	— 84.7
60	— 7.6	—12.1	—19.5	—31.2	—50.1	—80.4	—129.0

TABLE 39.—*The computation of the tangential velocities for each tube and altitude.*

Values of $\log v$.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	—	—	—	—	—	—	—
	170	0.76759	0.97305	1.17851	1.38397	1.58943	1.79489	2.00035
	160	0.91478	1.12024	1.32570	1.53116	1.73662	1.94208	2.14754
	150	0.99724	1.20270	1.40816	1.61362	1.81908	2.02454	2.23000
	140	1.05179	1.25725	1.46271	1.66817	1.87363	2.07909	2.28455
	130	1.08988	1.29534	1.50080	1.70626	1.91172	2.11718	2.32264
60°	120	1.11652	1.82198	1.52744	1.73290	1.93836	2.14382	2.34928
70	110	1.13425	1.33971	1.54517	1.75063	1.95609	2.16155	2.36701
80	100	1.14443	1.34989	1.55535	1.76081	1.96627	2.17173	2.37719
90	90	1.14776	1.35322	1.55868	1.76414	1.96060	2.17506	2.38052

Values of the tangential velocity, $v = Aa\varpi \sin az$.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	0	0	0	0	0	0	0
	170	5.9	9.4	15.1	24.2	38.9	62.4	100.1
	160	8.2	13.2	21.2	34.0	54.5	87.5	140.5
	150	9.9	16.0	25.6	41.1	65.9	105.8	169.8
	140	11.3	18.1	29.0	46.6	74.8	120.0	192.6
	130	12.3	19.7	31.7	50.9	81.6	131.0	210.2
60°	120	13.1	21.0	33.7	54.1	86.8	139.3	223.5
70	110	13.6	21.9	35.1	56.3	90.4	145.1	232.8
80	100	13.9	22.4	35.9	57.7	92.5	148.5	238.3
90	90	14.1	22.6	36.2	58.1	93.2	149.6	240.2

TABLE 40.—*The computation of the vertical velocities w, for each tube and altitude.*

Values of $\log w$.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	—∞	—∞	—∞	—∞	—∞	—∞	—∞
	170	8.73742	9.14834	9.55926	9.97018	0.38110	0.79202	1.20294
	160	9.03180	9.44272	9.85364	0.26456	0.67548	1.08640	1.49732
	150	9.19672	9.60764	0.01856	0.42948	0.84040	1.25132	1.66224
	140	9.30582	9.71674	0.12766	0.53858	0.94952	1.36044	1.77136
	130	9.38200	9.79292	0.20384	0.62476	1.03568	1.44660	1.85752
60°	120	9.43528	9.84620	0.25712	0.66804	1.07896	1.48988	1.90080
70	110	9.47074	9.88166	0.29258	0.70350	1.11442	1.52534	1.93626
80	100	9.49110	9.90202	0.31294	0.72386	1.13478	1.54570	1.95662
90	90	9.49775	9.90867	0.31959	0.73051	1.14143	1.55235	1.96327

Values of the vertical velocity, $w = 2A \sin az$.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$az = 180°$	0	0	0	0	0	0	0
	170	0.06	0.14	0.36	0.93	2.41	6.20	15.96
	160	0.11	0.28	0.71	1.84	4.74	12.20	31.43
	150	0.16	0.41	1.04	2.69	6.93	17.84	45.94
	140	0.20	0.52	1.34	3.46	8.90	22.93	59.07
	130	0.24	0.62	1.60	4.22	10.86	27.96	72.03
60°	120	0.27	0.70	1.81	4.66	11.99	30.90	79.58
70	110	0.30	0.76	1.96	5.05	13.01	33.52	86.35
80	100	0.31	0.80	2.06	5.30	13.64	35.13	90.49
90	90	0.32	0.81	2.09	5.38	13.85	35.67	91.89

THE HORIZONTAL ANGLE i AND VERTICAL ANGLE η OF THE CURRENT q IN THE ST. LOUIS TORNADO.

The horizontal angle i is directed inward from $az = 60°$ to $az = 90°$ and outward from $az = 90°$ to $az = 180°$. The angle i is calculated by the formula $\tan i = \frac{u}{v}$. Table 41 gives the value of i at 10° intervals for each tube.

TABLE 41.—*The horizontal angle i, negative inward, positive outward.*

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$az = 180°$	90°	90°	90°	90°	90°	90°	90°
170	80	80	80	80	80	80	80
160	70	70	70	70	70	70	70
150	60	60	60	60	60	60	60
140	50	50	50	50	50	50	50
130	40	40	40	40	40	40	40
120	30	30	30	30	30	30	30
110	20	20	20	20	20	20	20
100	10	10	10	10	10	10	10
90	0	0	0	0	0	0	0
80	—10	—10	—10	—10	—10	—10	—10
70	—20	—20	—20	—20	—20	—20	—20
60	—30	—30	—30	—30	—30	—30	—30

TABLE 42.—*Vertical angle η, positive upward.*

$$\tan \eta = \frac{w}{v \sec i}.$$

Altitude.		(1)		(2)		(3)		(4)		(5)		(6)		(7)	
	°	°	′	°	′	°	′	°	′	°	′	°	′	°	′
	$az=$ 180	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	170	0	6	0	9	0	14	0	23	0	37	0	59	1	35
	160	0	15	0	25	0	40	1	4	1	42	2	44	4	22
	150	0	27	0	44	1	10	1	52	3	0	4	49	7	42
	140	0	40	1	4	1	42	2	44	4	23	7	0	11	9
°	130	0	52	1	23	2	13	3	38	5	49	9	17	14	43
60	120	1	2	1	39	2	40	4	16	6	50	10	53	17	8
70	110	1	10	1	52	3	0	4	49	7	42	12	15	19	13
80	100	1	15	2	1	3	14	5	10	8	16	13	7	20	30
90	90	1	17	2	3	3	18	5	17	8	27	13	25	20	50

The total velocity q, in meters per second, can be computed from the formula

$$q = v \sec i \sec \eta$$

and the resulting values are given in Table 43.

TABLE 43.—*Total velocity q, in meters per second.*

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)
	°							
	$az=$ 180	∞	∞	∞	∞	∞	∞	∞
	170	33.72	54.12	86.86	139.42	223.76	359.17	576.57
	160	24.03	38.57	61.90	99.85	159.50	256.17	411.86
	150.	19.87	31.90	51.20	82.20	132.04	212.38	342.74
	140	17.53	28.14	45.17	72.54	116.64	188.05	305.32
°	130	16.06	25.78	41.39	66.51	107.08	173.25	283.71
60	120	15.10	24.25	38.94	62.60	100.91	163.75	270.06
70	110	14.50	23.28	37.39	60.14	97.06	157.97	262.38
80	100	14.16	22.74	36.53	58.78	94.94	154.94	258.38
90	90	14.06	22.57	36.26	58.34	94.26	153.84	257.14

We begin with $q = 15$ meters per second on the outer tube 1 and $q_2 = 25$ meters per second on the second tube, and it is seen that the total velocity on the lowest section of the St. Louis tornado reaches 164 meters per second on tube 6 and 270 meters per second on tube 7, near the axis. These are the velocities of the wind which caused the destructive effects in passing over the city. Chart X, fig. 7, gives the geometrical form of a section in the vertical plane $z\,\varpi$, of the tubes forming the St. Louis vortex. It shows that it was truncated at the plane $az = 60°$, and that in the topmost levels it overspread the base. These upper branches appear actually in nature as the turbulent cloud motions which precede and follow the storm center in the cumulus levels.

The volume of air V, in cubic meters per second, which passes upward thru each vortex tube, is computed from the formula,

$$V = \pi(\varpi_n^2 - \varpi^2_{n+1})\, w_n.$$

The results are shown in Table 44.

TABLE 44.—*Volume of air ascending in each vortex tube.*

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
°							
$az=$ 10	774500	774500	774500	774500	774500	774500	774500
80	774500	774500	774500	774500	774500	774500	774500
90	774500	774500	774500	774500	774500	774500	774500

This table shows that 774,500 cubic meters of air is passing upward thru each ring area per second. Since the Cottage City waterspout was carrying upward about 16,452 cubic meters of air per second, it follows that the *St. Louis tornado was about 47.08 times as efficient in lifting the air as was the Cottage City waterspout*, this being due to its greater dimensions.

EVALUATION OF THE FIRST EQUATION OF MOTION.

The pressure change on the horizontal plane is given by the first equation of motion, because the pressure has been assumed not to vary along the circles whose radii are ϖ_n. The full form of the equation is,

$$-\frac{\partial P}{\rho \partial \varpi} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial \varpi} + w\frac{\partial u}{\partial z} - \frac{v^2}{\varpi} - 2\,n\cos\theta\cdot v + ku,$$

$$= \frac{\partial u}{\partial t} + A^2 a^2 \varpi - 2\,n\cos\theta\cdot v + ku.$$

All these terms can be computed with precision, except the friction term ku, and some idea of the value of the friction coefficient may even be obtained.

The inertia term $\frac{\partial u}{\partial t}$.

In computing the inertia it is first necessary to find the time of the movement of the air between the successive rings, and the computation is given in full, as an example. The term can be found from any component, and that of the radial velocity is the most convenient to employ for this purpose. Find the mean u_m and the difference of the radii $\varpi_n - \varpi_{n+1}$ in succession, then,

$$\partial t = \frac{u_m}{\varpi_n - \varpi_{n+1}}. \tag{55}$$

Next compute in succession, $\partial u = u_n - u_{n+1}$, so that

$$\frac{\partial u}{\partial t} = \frac{u_m(u_n - u_{n+1})}{\varpi_n - \varpi_{n+1}}. \tag{56}$$

This is performed in Table 45.

TABLE 45.—*The computation of $\frac{\partial u}{\partial t}$.*

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\log u$	0.87796	1.08342	1.28088	1.49434	1.69980	1.90526	2.11072
$\log u_m$	0.98069	1.18615	1.39161	1.59707	1.80253	2.00799	
ϖ	960.0	598.0	372.7	232.2	144.7	90.2	56.2
$\varpi_n - \varpi_n+1$	361.9	225.4	140.5	87.5	54.5	34.0	
$\log(\varpi_n - \varpi_n+1)$	2.55859	2.35295	2.14768	1.94201	1.78640	1.53148	
$\log \partial t$	1.57790	1.16680	0.75607	0.34494	9.93387	9.52349	
∂t	37.84	14.68	5.70	2.21	0.86	0.334	
u	−7.6	−12.1	−19.5	−31.2	−50.1	−80.4	−120.0
$u_n - u_n+1$	4.5	7.4	11.7	18.9	30.3	48.6	
$\log(u_n - u_n+1$	0.65321	0.86923	1.06817	1.27646	1.48144	1.68644	
$\log \frac{\partial t}{\partial u}$	9.07531	9.70243	0.31212	0.93152	1.54757	2.16315	
$\frac{\partial t}{\partial u}$	0.119	0.504	2.052	8.541	35.283	144.60	

Similarly, the values of $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ are to be found as computed in Tables 46 and 47.

TABLE 46.—*Computation of the tangential inertia* $= \frac{\partial v}{\partial t}$.

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
v	13.1	21.0	33.7	54.1	86.8	139.3	223.5
$v_n - v_{n+1}$	7.9	12.7	20.4	32.7	52.5	84.2	
$\frac{\partial v}{\partial t}$	0.209	0.865	3.577	14.778	61.135	252.24	

TABLE 47.—*Computation of the vertical inertia* $= \frac{\partial w}{\partial t}$.

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
w	0.27	0.70	1.81	4.66	11.99	30.90	79.58
$w_n - w_{n+1}$	0.43	1.11	2.85	7.33	18.91	48.68	
$\frac{\partial w}{\partial t}$	0.011	0.076	0.500	3.318	22.020	145.83	

The deflecting forces.

The radial deflecting force, $-2n\cos\theta \cdot v_m$, and the tangential deflecting force, $+2n\cos\theta \cdot u_m$, are computed in Tables 48 and 49, using the mean values of v_m, u_m, computed thru their logarithms.

TABLE 48.—*Computation of the radial deflecting force,* $-2n\cos\theta \cdot v_m$.

$\phi = 38° 38'$. $\theta = 51° 22'$.

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log v	1.11652	1.32198	1.5-744	1.73290	1.93836	2.14882	2.34928
log $2n\cos\theta \cdot v$	7.07582	7.28128	7.48674	7.69220	7.89766	8.10312	8.30858
log $2n\cos\theta \cdot v_m$	7.17855	7.38401	7.58947	7.79493	8.00039	8.20585	
$-2n\cos\theta \cdot v_m$	—0.002	—0.002	—0.004	—0.006	—0.010	—0.016	

TABLE 49.—*Computation of the tangential deflecting force,* $+2n\cos\theta \cdot u_m$.

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log u	0.87796	1.08342	1.28888	1.49434	1.69980	1.90526	2.11072
$2n\cos\theta \cdot u$	0.001	0.001	0.002	0.004	0.006	0.009	

The fall in pressure between the successive vortex rings.

The fall in pressure, ΔP, between the successive rings ϖ_1, $\varpi_2, \ldots$, is computed by the formula,

$$\Delta P = \rho_m (\varpi_n - \varpi_{n+1}) \left[(A^2 a^2 \varpi)_m + \frac{\partial u}{\partial t} - 2n\cos\theta \cdot v_m + k u_m \right]$$

and this is performed in Table 49.

The result of the computation of $A^2a^2\varpi$ is given in Table 50, and all the other terms, except the friction coefficients, are brought together in the line marked Sum. This is to be multiplied by ρ_m $(\varpi_n - \varpi_{n+1})$ to give successive values of ΔP, the difference in pressure from one ring to another exprest in mechanical units. This is reduced to barometric pressure in millimeters by the formula,

$$\Delta B = \Delta P \times 0.0075.$$

If we again assume that the pressure on ring ϖ_1 is 737.0 millimeters, then the line marked B_c (Table 50) gives the barometric pressure as computed on the successive rings ϖ_1, ϖ_2, $\varpi_3, \ldots \varpi_7$. These values when plotted on a diagram, give a pressure-curve resembling a funnel-shaped vortex, which is apparently the shape of the pressure curve within the dumb-bell-shaped vortex. Altho we have no actual observations of pressure to guide us, it is yet possible to suppose that the observed pressures in the outer portions of the vortex decreased by 10 millimeters-differences on the rings ϖ_1, ϖ_2, ϖ_3, ϖ_4, giving 737, 727, 717, 707 millimeters in place of the computed pressures 737.0, 735.1, 730.2, 718.5. The differences +8.1, +13.2, +11.5, may possibly be due to the effect of friction on the tube, i. e., the tube in passing over the city, may be distorted in its lowest section by its work of destruction on the houses. Those values of $\Delta B = B_c - B_0$ reduced to ΔP, become 1080, 1760, 1533. Now the value of the coefficient of friction, k, should be found from,

$$k = \frac{\Delta P}{u_m (\varpi_n - \varpi_{n+1})}, \tag{57}$$

and for the mean value of $\Delta P = 1458$, we find $k = 0.2867$. This value for the coefficient of friction may not be very exact, because we lack observed values of B, but it illustrates a method of computing k which can be applied in the study of hurricanes, ocean and land cyclones.

TABLE 50.—*Computation of the fall in pressure,* ΔP, *between the successive vortex rings.*

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$A^2 a^2 \varpi$	0.238	0.982	4.059	16.782	69.380	286.82	1185.75
$(A^2 a^2 \varpi)_m$	0.483	1.997	8.254	34.122	141.07	583.18	
$\frac{\partial u}{\partial t}$	0.119	0.504	2.052	8.541	35.28	145.60	
$-2n\cos\theta \cdot v$	—0.002	—0.002	—0.004	—0.006	—0.010	—0.02	
Sum	0.600	2.499	10.302	41.657	176.44	728.76	
log Sum	9.77815	0.39777	1.01292	1.62999	2.24635	2.86259	
ϖ_n	960.0	598.1	372.7	232.2	144.7	90.2	56.2
$\varpi_n - \varpi_n + 1$	361.9	225.4	130.5	87.5	54.5	34.0	
log $(\varpi_n - \varpi_n + 1$	2.55859	2.35295	2.11561	1.94201	1.73640	1.53148	
ρ_m	0.06608	0.06608	0.06608	0.06608	0.06608	0.06608	
log ΔP	2.40282	2.81680	3.19461	3.63808	4.04883	4.46015	
log ΔB	0.27788	0.69186	1.06967	1.51314	1.92389	2.33521	
ΔB (in mm.)	1.90	4.92	11.74	32.59	83.92	216.38	
FRICTION COEFFICIENTS.							
B_c	737.0	735.1	730.2	718.5	685.9	602.0	385.6
B_o	737.0	727.0	717.0	707.0	697.0	687.0	677.0
$B_c - B_o$	0.0	+8.1	+13.2	+11.5	—11.1	—85.0	—291.4
ΔP	....	1080	1760	1533	1480	....	....
$u_m (\varpi_n - \varpi_n + 1) \rho_m$	....	5106	5104	5106	5104	....	....
k	....	0.2115	0.3448	0.3003	0.2900	....	....

THE CAUSE OF THE DESTRUCTIVE EFFECTS IN THE ST. LOUIS TORNADO.

After the passage of the tornado over the city of St. Louis it was found that immensely powerful forces of destruction had been in operation. Trees had been uprooted, their tops had been twisted off at the trunk, large buildings had been wrecked in every conceivable way, heavy stones and irons had been moved bodily, iron girders had been twisted and torn, a plank had been driven thru the webbing of a steel girder of the bridge, and numberless instances of powerful forces in operation are on record. We can compute[1] the values of the pressure differences between successive vortex rings from the formula,

$$\Delta B = 0.001742 \frac{B}{T} q^2, \tag{58}$$

where ΔB is the pressure difference exprest in millimeters, T is the absolute temperature, and q is the wind velocity in meters per second.

The resulting computation and values of ΔB are given in Table 51.

It was observed that the destructive effects of the tornado seem to diminish greatly at a plane about 30 feet above the ground, the second and upper stories of the buildings suffering much more than the first story. This implies that the vertical forces in the tube were cut off at that plane by the disturbing frictional resistances of the rough surface of the city. An inspection of the pressures developed by the wind having a velocity q, shows that in the center of the vortex the pressure can, theoretically, run up to about 8,000 pounds per

[1] See Monthly Weather Review, October, 1906, XXXIV, p. 470, formula (11).

square foot, and the wind velocity can reach 270 meters per second, or 600 miles per hour. Whatever may be thought as to the actual development of such velocities and forces in this tornado, it is evident that sufficient power has been revealed to account fully for all the mechanical forces that were observed and considered by engineers. Mr. Julius Baier thought that something like 100 pounds per square foot had been expended in the destructive effects, but it is evident that much greater forces were really available near the center of the tube. At some distance from the center, in the tubes $\varpi_3=375$ meters to $\varpi_4=240$ meters, the pressures were apparently from 175 to 450 pounds per square foot. The subordinate minor whirls, or small vortices caused by the wind twining around obstacles, builds up the so-called frictional coefficient. In the free air the value of k is apparently a negligible quantity, and large values of k are confined to a thin surface layer.

TABLE 51.—*Approximate pressure in pounds per square foot exerted by the wind in the St. Louis tornado.*

$$\Delta B = 0.001742 \frac{B}{T} q^2.$$

Tubes.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
q	15.10	24.25	38.94	62.60	100.91	163.75	270.06
B	0.737	0.727	0.717	0.707	0.697	0.689	0.677
T	294.	294.	294.	294.	294.	294.	294.
ΔB (mm.)	9.96	25.33	64.42	164.16	420.33	1091.5	2925.6
$\Delta B \left(\frac{\text{pounds}}{\text{foot}^2}\right)$	27.66	70.37	178.96	456.03	1167.7	3032.1	8127.2

1 mm. mercury = 2.778 pounds per square foot.

must take on an additional velocity as soon as the cold layer is placed upon it. Now, in the St. Louis tornado a cold mass of air was carried forward over the warm mass of stagnant air that had been lying over the city for several days, and in a few hours the temperatures fell about 18° F. = 10° C., the tornado occurring at the vertical junction of two masses of air at different temperatures, as in Table 30. It seems probable that *the warm air instead of mixing vertically with the cold sheet, slid out horizontally in all directions, that is radially from the point of greatest temperature contrast*, like the spokes of a wheel held horizontally above the head. If the velocities u, v, w in Tables 38, 39, and 40 are examined at the higher sections $az = 180°$, or $az = 170°$, it is seen that in this vortex the radial velocity above survives. Hence, we infer that the cause of this tornado was the horizontal flow of the warm air away from a center under the cold overflowing sheet, and that this radial action, whose purpose is to counteract the pressure change brought by the overflowing cold sheet, then propagated itself vertically downward in a dumb-bell-shaped vortex till it was cut off by the rough surface of the country and city at a section corresponding to an inflow of $i = 30°$, as found in the observations. This example of the effects of horizontal convection suggests the forces which are operating in the atmosphere during the mixture of warm or cold currents. Similar reasoning assigns the same cause for the generation of hurricanes, which are deep tornadoes of the dumb-bell shape (see fig. 8). The same action can be traced to about half the area of large ocean cyclones, but the inner rings show that the horizontal convection is due in part to the sheets of cold and warm air standing vertically, while in the land cyclone the

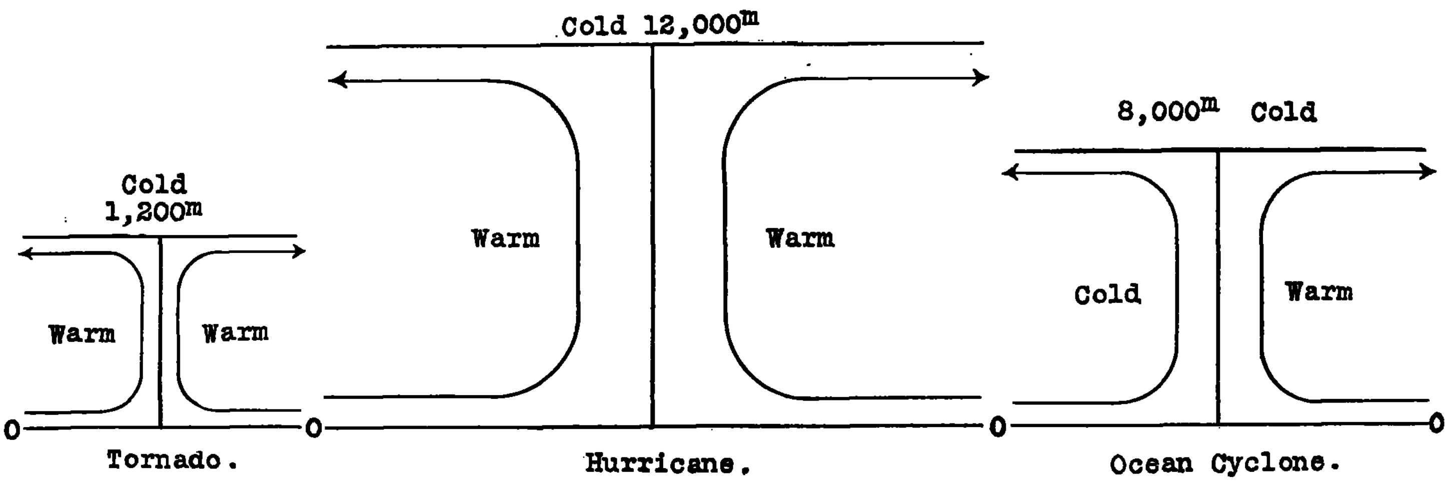

FIG. 8.—Diagram of the cold and warm masses of air in the tornado, hurricane, and ocean cyclone.

vertical position of the plane separating the warm air from the cold air prevails and gives very impure vortices, tho their general typical features still survive.

The hurricane will be illustrated by the De Witte typhoon of August 1-3, 1901.

THE CAUSE OF THE FORMATION OF THE ST. LOUIS TORNADO.

In the last section of the first paper of this series on vortex motions it was shown that when two masses of air of different temperatures overlay one another, as a cold layer over a warm layer, there is a discontinuity in the pressure, caused by the different densities. But since in the air these discontinuities in the pressure can not persist under the forces of gravity, there is an immediate setting up of certain currents of motion which tend to destroy these pressure discontinuities and to restore a simple pressure gradient, such as is consistent with the prevailing temperatures. These temperatures and velocities are connected by the formula,

$$T_2(v_1^2 - v_0^2) = T_1(v_2^2 - v_0^2)$$

in which T_1 and v_1 are the temperature and average velocity, respectively, of the warm layer and T_2 and v_2 are the temperature and velocity of the cold layer and v_0 is the average velocity of the layer before disturbance.

Since the temperature of the cold layer, T_2, is connected with the motion of the warm layer, it follows that the warm layer

A TWO YEARS' STUDY OF SPRING FROSTS AT WILLIAMSTOWN, MASS.

By Prof. WILLIS I. MILHAM, Ph. D. Dated Williamstown, Mass., August 11, 1908.

INTRODUCTION.

Spring frosts have been quite extensively studied, chiefly on account of the damage caused by them which has excited popular interest in their prediction and in methods of protection against them. Among the more recent articles by those connected with the U. S. Weather Bureau may be mentioned:

Cline, I. M., "Irregularities in Frost and Temperature in Neighboring Localities." Third Convention of Weather Bureau Officials, Proceedings. Washington, D. C., 1904, p. 250.

Garriott, E. B., "Notes on Frost." Farmers Bulletin, No. 104.